Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Mathematical Methods-II

Subject Code: 5SC04MBE1 **Branch: M.Sc. (Mathematics)**

Semester: 4 **Date: 26/04/2017** **Time: 10:30 To 01:30** **Marks: 70**

<u>Instructions</u>:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Answer the Following questions: **(07)**

a. If a functional $I[y(x)] = \int_0^1 y(x)\,dx$ is defined on the class $C[0,1]$, then prove that $I\left[\cos\left(\frac{\pi x}{2}\right)\right] = \frac{1}{\pi}$. **(02)**

b. Solve the Euler's equation of the functional $I[y(x)] = \int_{x_0}^{x_1}(x + y')y'\,dx$. **(02)**

c. Show that $y(x) = 1$ is a solution of $\int_0^x y(t)\,y(x-t)\,dt = 2y + x - 2$. **(02)**

d. The shortest distance between two fixed points in the Euclidean xy-plane is a straight line. Determine whether the statement is True or False. **(01)**

Q-2 Attempt all questions **(14)**

a. Show that **(07)**

$$\int_a^x \int_a^{x_n} \dots \int_a^{x_2} f(x_1)dx_1 dx_2 \dots dx_n = \frac{1}{(n-1)!}\int_a^x (x-t)^{n-1} f(t)\,dt$$

b. Show that the curve which extremizes the functional $\int_0^{\frac{\pi}{4}}(y''^2 - y^2 + x^2)\,dx$ under the conditions $y(0) = 0, y'(0) = 1, y\left(\frac{\pi}{4}\right) = y'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ is $y(x) = \sin x$. **(07)**

OR

Q-2 Attempt all questions **(14)**

a. Show that the geodesics on a sphere of radius $'a'$ are its great circle. **(07)**

b. Find the integral equation corresponding to boundary value problem **(07)**
$y'' + \lambda\, y(x) = 0, y(0) = y(1) = 0.$

Q-3 Attempt all questions **(14)**

a. On what curve the functional $\int_0^{\pi/2} (y'^2 - y^2 + 2xy)\, dx$ with $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$ be extremized? **(05)**

b. Find the extremal of the functional $\int_0^{\pi} (y'^2 - y^2)\, dx$ under the conditions $\int_0^{\pi} y\, dx = 1,\ y(0) = 0$ and $y(\pi) = 1$. **(05)**

c. Let a functional $I[y(x)]$ defined on the class $C^1[0,1]$ be given by **(04)**
$I[y(x)] = \int_0^1 [1 + \{y'(x)\}^2]^{1/2}\, dx$. Then prove that $I[1] = 1, I[x] = \sqrt{2}$ and $I[x^2] = \frac{\sqrt{5}}{2} + \frac{1}{4}\sinh^{-1} 2$.

OR

Q-3 Attempt all questions **(14)**

a. In usual notations prove that **(05)**

$(i)\ \frac{d}{dx}\left(F - y'\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial x} = 0,$

$(ii)\ \frac{\partial F}{\partial y} - \frac{\partial^2 F}{\partial x\, \partial y'} - y'\frac{\partial^2 F}{\partial y \partial y'} - y''\frac{\partial^2 F}{\partial y'^2} = 0\ .$

b. Find the extremals of the functional $\int_0^{\pi} (4y \cos x + y'^2 - y^2)\, dx$ that satisfy the given boundary conditions $y(0) = y(\pi) = 0$. **(05)**

c. Show that the functional $I_1[y(x)] = \int_a^b \{y'(x) + y(x)\}\ dx$ is linear in the class $C^1[a,b]$, but the functional $I_2[y(x)] = \int_a^b [p(x)\{y'(x)\}^2 + q(x)\{y'(x)\}^2]\ dx$ is non-linear. **(04)**

SECTION - II

Q-4 Answer the Following questions: **(07)**

a. Reduce the differential equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ in to Sturm-Liouville differential equation. **(02)**

b. Write the Chebyshev and Laguerre differential equation. **(02)**

c. Obtain the solution of $y(x) = 1 + \lambda \int_0^1 x\, t \cdot\ y(t)\, dt$ in the form $y(x) = 1 + \frac{3\lambda x}{2(3-\lambda)}\ (\lambda \neq 3)$. **(02)**

d. Define: Degenerate kernel. **(01)**

Q-5 Attempt all questions **(14)**

a. Find the eigenvalues and eigenfunctions of **(07)**

$$[xy']' + \left(\frac{\lambda}{x}\right) y = 0, y'(1) = y'(e^{2\pi}) = 0.$$

b. Discuss the eigenvalues and corresponding eigenfunctions of the integral equation $y(x) = F(x) + \lambda \int_0^1 (1 - 3xt)y(t)\, dt$. **(07)**

OR

Q-5 Attempt all questions **(14)**

a. Find the all eigenvalues and eigenfunctions of the Strum-Liouville problem **(07)**

$$[x^3 y']' + \lambda xy = 0, y(1) = y(e) = 0.$$

b. Solve the integral equation $F(t) = 1 + \int_0^t F(u) \sin(t-u)\, du$ and verify your solution. **(07)**

Q-6 Attempt all questions **(14)**

a. Convert the integral equation $y(x) = 6x - 5 + \int_0^x (5 - 6x + 6t)\, y(t)\, dt$ in to differential equation along with initial conditions. **(05)**

b. Solve the integral equation $\int_0^x \frac{y(t)}{\sqrt{x-t}}\, dt = 1 + 2x - x^2$. **(05)**

c. Find the eigenvalue and eigenfunction of the integral equation $y(x) = \lambda \int_0^1 e^{x+t}\, y(t)\, dt$. **(04)**

OR

Q-6 Attempt all Questions **(14)**

a. Solve the integral equation $y(x) = x + \lambda \int_0^1 (x-t)y(t)dt$. **(05)**

b. Solve the integral equation $F'(t) = t + \int_0^t F(t-u) \cos u\, du$, if $F(0) = 4$. **(05)**

c. Reduce the differential equation $4x^2 y'' + 4xy + (64x^2 - 9)y = 0$ to Sturm-Liouville differential equation. **(04)**